

## **Optimal Multi Antenna Spectrum Sensing Technique For Cognitive Radio**

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### **ABSTRACT**

In this paper, we consider the primary user detection problem in cognitive radio systems by using multi antenna at the cognitive radio receiver. An optimal square law combiner multi antenna based spectrum sensing technique is proposed using the multitaper spectrum estimation method. The multitaper spectrum estimation method (MTM) produces single spectrum estimate with minimum spectral leakage and variance using an orthonormal family of tapers, the Discrete Prolate Slepian Sequences (DPSS). An energy detector (ED), which is simple, but has a poor performance at a low signal to noise ratio (SNR).

**Keywords:** Cognitive radio, spectrum sensing, multi taper spectrum estimation method, multi antenna spectrum sensing.

### **I. INTRODUCTION**

Radio Spectrum refers to the existing, natural medium that is used in different wireless communication systems and services: mobile, fixed, satellite-based, and low-power device communications systems (ultra wideband, sensors etc.)[1]. Due to increase in wireless technologies, there is a problem of spectrum scarcity. Cognitive radio (CR) is a new wireless communication technology that assigns a spectrum dynamically to the secondary users when the primary users are not using licensed spectrum.

There are different types of spectrum sensing techniques are there like Matched filter, Energy detector, cyclostationary, multi taper method. Matched filter [2] and cyclostationary are classified as high performance spectrum sensing techniques but they require every primary user transmitted signal i.e. which requires primary user information. The disadvantage in the cyclostationary detection [3] is which requires cyclic frequencies of the primary user and also it takes long sensing time. Similarly the practical implementation of Energy detector is easy but gives very poor performance at low signal to noise ratio (SNR).

### **II. IMPORTANCE OF MTM METHOD FOR A SPECTRUM SENSING IN COGNITIVE RADIO NETWORK**

Multitaper spectrum estimation method (MTM) was proposed in 1982 by Thomson [4]. MTM uses an optimal bank of band pass filters (known as tapers or windows). These orthonormal tapers are called Discrete Prolate Slepian Sequences (DPSS) [6]. MTM produces a single spectrum estimate with minimum spectral leakage and good variance. The spectrum estimation in MTM is an approximation of the optimal estimate; the maximum likelihood (ML). One advantage for MTM compared to ML is the fact that it has lower computation complexity. Since the first development of MTM in 1982, this advanced method has been widely used in many applications. In addition to the power spectrum estimation in signal processing and communications applications, MTM is used in neurosciences, geophysics and sonar.

### **III. THE IMPORTANCE OF USAGE OF MULTI ANTENNA BASED SPECTRUM SENSING IN CR SYSTEMS**

In order to improve the spatial diversity, classical wireless communications use multi antenna at transmitter (Tx) or receiver (Rx) or both. Such diversity improvement increases the system data rate

and capacity. The reason behind this is that as the distance between antennas is chosen properly there will be a high probability of receiving independent fading through these different antennas [9]. Therefore, the fading effects will be mitigated.

#### IV. ENERGY DETECTOR

The probability of detection  $P_d$  is defined as is the probability that the CR detector decides correctly the presence of the PR's signal. Therefore, this type of probability is related to the  $P(D; H_1)$  which represents the PR's signal plus noise

$$P_d = P_r \{D > \gamma / H_1\} = \int_{\gamma}^{\infty} P(D; H_1) dt \quad (1)$$

where  $P_r$  represents the probability. It is clear

that  $p_d$  is the integration over  $P(D; H_1)$  from the threshold  $\gamma$  to infinity. Since the  $P(D; H_1)$  is assumed as Gaussian distribution;

$P_d$  can be defined finally as follows[5]

$$P_d = Q\left(\frac{\gamma - E[p(D; / H_1)]}{\sqrt{\text{VAR}[P(D; H_1)]}}\right) \quad (2)$$

where E and Var, are the mean and variance of the given distribution respectively. The term  $Q(\xi)$  is the complementary cumulative distribution function,

$$Q(\xi) = \frac{1}{\sqrt{2\pi}} \int_{\xi}^{\infty} e^{-\frac{t^2}{2}} dt \quad (3)$$

The probability of false alarm,  $P_f$  is defined as the probability that the CR detector decides by mistake the presence of the PR's signal. Therefore, this type of probability is related to the  $P(D; H_0)$  which represents the noise only.

$$P_f = P_r \{D > \gamma / H_0\} = \int_{\gamma}^{\infty} P(D; H_0) dt \quad (4)$$

In this case, it is clear that the  $P_f$  is the integration over  $P(D; H_0)$  from the threshold  $\gamma$  to infinity[5]

$$P_f = Q\left(\frac{\gamma - E[p(D; / H_0)]}{\sqrt{\text{VAR}[P(D; H_0)]}}\right) \quad (5)$$

where E and Var, are the mean and variance of the given distribution respectively.

#### V. MULTI TAPER SPECTRUM ESTIMATION METHOD

some of the advantages of multi taper estimation method are

1. MTM is an energy based spectrum sensing technique
2. MTM is a wideband spectrum sensing technique
3. MTM does not need any prior information about the PR's signal (i.e., non coherent nor partial coherent).
4. MTM needs to know the noise variance to control the threshold.
5. MTM minimizes the spectral leakage outside the band and improves the variance of estimate.

A general procedure for multi taper spectrum estimation consists of four steps. First, choose a time-bandwidth product  $C_0 = NW$  where N is the sample size for estimation, and W is the bandwidth normalized by the sample rate. The choice of parameter  $C_0$  is a trade-off between spectral resolution and variance. Typically,  $3 \leq C_0 \leq 6$ , and we use  $K = 2C_0 - 1$  data tapers to perform the estimation for the reason that will be given below. Second, compute the data tapers, i.e., Slepian sequences.

Denote the  $k$ th data taper in vector form by  $w_k$ . It can be calculated by the following Eigen equation

$$KW_k = \lambda W_k \quad (6)$$

where the  $(i, j)$  th entry of Toeplitz matrix K is defined by

$$K_{i,j} = \frac{\sin 2\pi w(i-j)}{\pi(i-j)} \quad i, j = 1, 2, 3, \dots, N \quad (7)$$

The Eigen values  $\lambda_k$  range between 0 and unity, and are organized in an descending order such that  $\lambda_1 \geq \lambda_2 \geq \dots \lambda_N$ . The first

$K \approx [2NW]$  of them are dominated, close to unity, whereas the rest are negligible. Moreover, the tapers of lower order have much stronger energy-concentration capability than their high-order counterparts, suggesting that it suffices to use the first  $k$  tapers for spectral estimation. Third, the corresponding eigenspectra are defined by Fourier transform of the tapered (i.e., windowed) data sequences, resulting in

$$Y_k(f) = \sum_{n=1}^N w_k(n) x(n) e^{-j2\pi f n} \quad k = 0, 1, 2, \dots, K-1 \quad (8)$$

as a function of frequency  $f$ . Here,  $w_k(n)$  signifies the  $n$ th entry of  $w_k$ . Given a finite sample-size constraint, the energy distribution of each eigenspectrum falls between the bandwidth from  $f - W$  to  $f + W$ , due to

energy-concentration property of Slepian sequences. Finally, we combine the Eigen spectra to form a single spectrum estimate,

$$s^{\wedge}(f) = \sum_{k=0}^{K-1} \alpha_k |Y_k(f)|^2 \quad (9)$$

where the weighting factor  $\alpha_k = \lambda_k / (\lambda_0 + \lambda_1 + \dots + \lambda_{K-1})$  is used to account for the relative importance of the Eigen spectrum of concern.

The received PR signal, at the CR receiver, is sampled to generate a finite discrete time samples series  $\{x_{t,m}; t = 0, 1, 2, \dots, N-1, m = 1, 2, \dots, M\}$ , where  $m$  denotes the antenna number and  $t$  is the time index. The discrete time samples are dot multiplied with different tapers  $v_{t,k}(N, W)$  (Tapers are DPSS).

The associated Eigen value of the  $k^{th}$  taper is  $\lambda_k(N, W)$ . The product is applied to a Fourier transform to compute the energy concentrated in the bandwidth  $(-W, W)$  centred at frequency,  $f$ . The half time bandwidth product is  $NW$ . The total number of generated tapers is  $2NW$ .

The binary hypothesis test for CR spectrum sensing at the  $l^{th}$  time, and using the  $m^{th}$  antenna branch is given by

$$\begin{aligned} H_0 : x_{t,m}(l) &= w_{t,m}(l) \\ H_1 : x_{t,m}(l) &= s_t(l) + w_{t,m}(l) \end{aligned} \quad (10)$$

where  $l=0, 1, 2, \dots, L-1$  OFDM block's index, and  $x_{t,m}(l)$ ,  $w_{t,m}(l)$  and  $s_t(l)$  denote the CR received, noise at the branch  $m$  and PR's transmitted samples.

The transmitted PR signal is distorted by the zero mean AWGN,  $w_{t,m}(l) \approx N(0, \sigma_w^2)$  at the output from the different antenna branches, which are independent and with identical variance. For  $K$  orthonormal tapers used in the MTM, there will be  $K$  different Eigen spectrums produced from each antenna defined as [6]

$$Y_{k,m}(f_i) = \sum_{t=0}^{N-1} v_{(t,k)}(N, W)(x_{t,m}(l))e^{-j2\pi f_i t} \quad (11)$$

where  $f_i = 0, \frac{1}{N}, \frac{2}{N}, \frac{3}{N}, \dots, \frac{N-1}{N}$  are the normalized frequency bins.

The power spectrum estimate given by [4]

$$s_{MTM}^m(f_i) = \frac{\sum_{k=0}^{K-1} \lambda_k(N, W) |Y_{k,m}(f_i)|^2}{\sum_{k=0}^{K-1} \lambda_k(N, W)} \quad (12)$$

The energy detector, when the samples are taken at uniform time spacing, gives the power spectrum density estimation[7]

$$s_{ED}^m(f_i) = \left| \sum_{t=0}^{N-1} x_{t,m}(l) e^{-j2\pi f_i t} \right|^2 \quad (13)$$

A) Decision statics for the MTM and ED

The decision statistic over  $L$  using MTM is defined for the  $m^{th}$  antenna as

$$DEC_{MTM}^m(f_i) = \sum_{l=0}^{L-1} \frac{\sum_{k=0}^{K-1} \lambda_k(N, W) \left| \sum_{t=0}^{N-1} v_{t,k}(N, W) x_{t,m}(l) e^{-j2\pi f_i t} \right|^2}{\sum_{k=0}^{K-1} \lambda_k(N, W)} \quad (13)$$

The decision statistic over  $L$  using ED is defined for the  $m^{th}$  antenna as

$$DEC_{ED}^m(f_i) = \sum_{l=0}^{L-1} \left| \sum_{t=0}^{N-1} x_{t,m}(l) e^{-j2\pi f_i t} \right|^2 \quad (14)$$

B) Mean and Variance of MTM and ED using Single Antenna

For single antenna MTM-based spectrum sensing, and according to the central limit theorem, if the number of samples  $L$ , is large, the decision statistic,  $DEC_{MTM}^m(f_i)$  is asymptotically normally distributed with mean[8]

$$E[DEC_{MTM}^m(f_i)] = \begin{cases} LK\sigma_w^2 & H_0 \\ LK(E_s + \sigma_w^2) & H_1 \end{cases} \quad (15)$$

and variance (VAR)

$$VAR(DEC_{MTM}^m(f_i)) = \begin{cases} 2LK^2\lambda_2\sigma_w^4 & H_0 \\ 2LK^2\lambda_2\sigma_w^2(\sigma_w^2 + 2E_s) & H_1 \end{cases} \quad (16.1)$$

$$c = E \left[ \frac{1}{\sum_{k=0}^{K-1} \lambda_k(N, W)} \right] = \frac{1}{\sum_{k=0}^{K-1} \lambda_k(N, W)} \quad (17)$$

The decision statistic, for the Energy Detector  $DEC_{ED}^m(f_i)$  is asymptotically normally distributed with mean[9] (18)

and variance (VAR)

$$VAR(DEC_{ED}^m(f_i)) = \begin{cases} 2L\sigma_w^4 & H_0 \\ 2L\sigma_w^2(\sigma_w^2 + 2E_s) & H_1 \end{cases} \quad (19)$$

C) Detection and False Alarm probabilities

For a normally distributed decision statistic,  $DEC(f_i)$  the probabilities of detection  $P_d(f_i)$  and false alarm,  $P_f(f_i)$  are defined as

$$P_d(f_i) = P(DEC(f_i) > \gamma / H_1) = Q\left(\frac{\gamma - E(DEC(f_i) / H_1)}{\sqrt{VAR(DEC(f_i) / H_1)}}\right) \quad (20)$$

$$P_f(f_i) = P(DEC(f_i) > \gamma / H_0) = Q\left(\frac{\gamma - E(DEC(f_i) / H_0)}{\sqrt{VAR(DEC(f_i) / H_0)}}\right) \quad (21)$$

The probability of miss detection  $P_{mi}(f_i)$  is defined as

$$P_{mi}(f_i) = P(DEC(f_i) < \gamma / H_1) = 1 - Q\left(\frac{\gamma - E(DEC(f_i) / H_1)}{\sqrt{VAR(DEC(f_i) / H_1)}}\right) \quad (22)$$

The term  $Q(\xi)$  is the complementary cumulative distribution function  $Q(\xi) = \frac{1}{\sqrt{2\pi}} \int_{\xi}^{\infty} e^{-\frac{t^2}{2}} dt$  and  $\gamma$  represents the chosen threshold.

Thus, the different probabilities of MTM using single antenna can be redefined as [8]

$$P_d^{MTM}(f_i) = Q\left(\frac{\gamma - LK(E_s + \sigma_w^2)}{\sqrt{2LC^2\lambda_{\Sigma}\sigma_w^2(\sigma_w^2 + 2E_s)}}\right) \quad (23)$$

$$E[DEC_{ED}^m(f_i)] = \begin{cases} L\sigma_w^2 & H_0 \\ L(E_s + \sigma_w^2) & H_1 \end{cases} \quad (24)$$

$$P_f^{MTM}(f_i) = Q\left(\frac{\gamma - LK\sigma_w^2}{\sqrt{2LC^2\lambda_{\Sigma}\sigma_w^4}}\right) \quad (25)$$

$$P_{mi}^{MTM}(f_i) = 1 - Q\left(\frac{\gamma - LK(E_s + \sigma_w^2)}{\sqrt{2LC^2\lambda_{\Sigma}\sigma_w^2(\sigma_w^2 + 2E_s)}}\right) \quad (26)$$

The number of samples required by MTM using single antenna ( $L^{MTM}$ ) can be written as [8]

$$L^{MTM} = \left(\frac{a^{MTM} - b^{MTM}}{KE_s}\right)^2 \quad (27)$$

where a and b are

$$a^{MTM} = \sqrt{2LC^2\lambda_{\Sigma}\sigma_w^4} Q^{-1}(P_f^{MTM}(f_i)) \text{ and}$$

$$E[DEC_{MTM-SLC}(f_i)] = \begin{cases} MLK\sigma_w^2 & H_0 \\ MLK(E_s + \sigma_w^2) & H_1 \end{cases}$$

$$b^{MTM} = \sqrt{2LC^2\lambda_{\Sigma}\sigma_w^2(\sigma_w^2 + 2E_s)} Q^{-1}(P_d^{MTM}(f_i)).$$

$$E[DEC_{ED-SLC}(f_i)] = \begin{cases} ML\sigma_w^2 & H_0 \\ ML(E_s + \sigma_w^2) & H_1 \end{cases}$$

D) Mean and Variance of MTM and ED using M Antennas

The decision statistics for square law combining using M antennas for both techniques MTM, and ED respectively as follow [12]

$$DEC_{MTM-SLC}(f_i) = \sum_{m=1}^M \sum_{l=0}^{L-1} \frac{\sum_{k=0}^{K-1} \lambda_k(N,W) \left| \sum_{t=0}^{N-1} v_{t,k}(N,W) x_{t,m}(l) e^{-j2\pi f t} \right|^2}{\sum_{k=0}^{K-1} \lambda_k(N,W)} \quad (28)$$

and

$$DEC_{ED-SLC}(f_i) = \sum_{m=1}^M \sum_{l=0}^{L-1} \left| \sum_{t=0}^{N-1} x_{t,m}(l) e^{-j2\pi f t} \right|^2 \quad (29)$$

The decision statistic using square law combining is a sum of identical and independent normally distributed M antennas decision statistics. Thus  $DEC_{MTM-SLC}(f_i)$  the mean of the using M antennas [13] can be defined as (30) and the variance

$$VAR(DEC_{MTM-SLC}(f_i)) = \begin{cases} 2MLC^2\lambda_{\Sigma}\sigma_w^4 & H_0 \\ 2MLC^2\lambda_{\Sigma}\sigma_w^2(\sigma_w^2 + 2E_s) & H_1 \end{cases} \quad (31)$$

For the ED, the mean of the decision statistic, using square law combining through M antennas can be defined as (32)

## VI. SIMULATION RESULTS

In our system each node of the cognitive radio (CR) network uses 64-FFT with sampling frequency 20MHz. The primary user (PR) transmitter uses 64-IFFT with symbol duration  $T_s=0.05\mu s$ , and transmits QPSK signal with normalized equal to 1 with each subcarrier. The multi antenna technique MTM-SLC, ED-SLC. In MTM techniques the used half-time and width product is  $NW=4$ , and the number of tapers  $k=5$ . In fig.1 and fig 2 consider the AWGN channel with  $SNR=-10dB$  and 20 OFDM blocks (i.e  $L=20$ ) used in sensing and the number of samples used is  $(L=20) \times (N=64) = 1280$ , which approximately corresponds to  $64 \mu s$ .

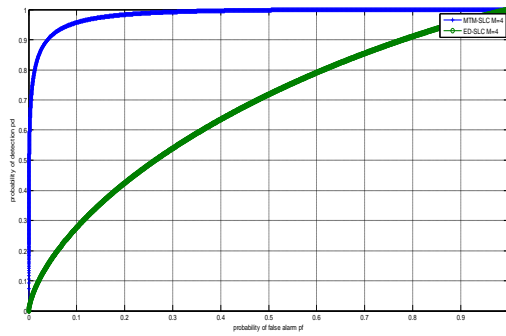


Fig. 1. Probability of detection Vs probability of false alarm using MTM and ED with M=4 Antennas at the Receiver.

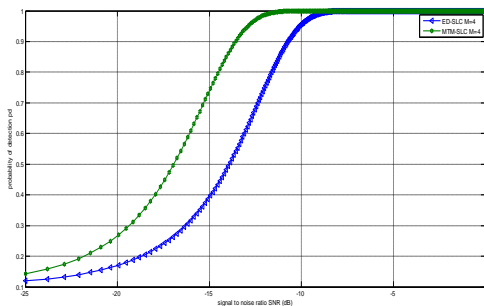


Fig. 2. Signal to noise ratio (SNR) Vs probability of detection for M=4 Antennas at the Receiver.

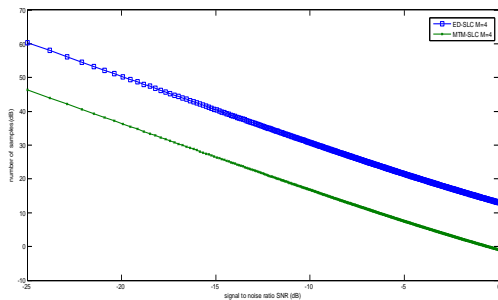


Fig. 3. Signal to Noise ratio (SNR) Vs Number of samples (dB) for M=4 Antennas at the Receiver.

## VII. CONCLUDING REMARKS

Here we consider the detection of primary user by taking MTM and ED detectors. Multiple antennas (M=4) used at the receiver side. Using multi antenna in MTM-SLC gives more improvement in performance compared to that for ED-SLC and also MTM method requires less number of samples to detect the primary user compared to the ED. The OR rule cooperation is used to cooperate the generated binary decisions from the individuals CR nodes at a

main CR-BS. Then the final decision is declared to the CR nodes.

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Where  $\lambda_{\Sigma}$  is

$$\lambda_{\Sigma} = \sum_{k=0}^{K-1} \lambda_k^2(N, W) + 2\lambda_0(N, W)\lambda_1(N, W) + 2\lambda_0(N, W)\lambda_2(N, W) + \dots + 2\lambda_0(N, W)\lambda_{K-1}(N, W) + 2\lambda_1(N, W)\lambda_2(N, W) + \dots + 2\lambda_1(N, W)\lambda_{K-1}(N, W) + \dots + 2\lambda_{K-2}(N, W)\lambda_{K-1}(N, W) \quad (16.2)$$